A Brief Introduction to Euclidean Geometry in Competition Math

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April 2020

Problems

1. Find the area of the largest triangle that can be placed inside a square with sidelength 1.
2. Let $ABCD$ be a square with sidelength 1. Square $EFGH$ is created by connecting the midpoints of the square formed by the midpoints of $ABCD$. Find the value of
$$\frac{[EFGH]}{[ABCD]}$$
given that $[X]$ denotes the area of figure X.
3. Let ω be a circle with radius $k \in \mathbb{R}$. Points A, B, C, D are on the circle in counterclockwise order such that $ABD = \angle 37^\circ$. The smaller angle formed by the diagonals is also 37°. Find $\angle BCD$ in degrees.

4. Find the area of a regular octagon with sidelength 1.
5. Triangle ABC has an area of 12. Point D lies on BC such that
$$BD = 2 \cdot DC.$$
Find the area of triangle ADB.
6. A regular hexagon has a sidelength of k. Find all possible values of k given that the hexagon has an integer area given that $k \in \mathbb{R}^+$.

7. Quadrilateral $ABCD$ has perpendicular diagonals with integer lengths. Find the maximum possible area of $ABCD$ given that $AB = 13$ and
$$AB > BC > CD > DA.$$
8. Let $a_1, a_2, \ldots, a_{201}$ be 201 equidistant points on a circle. Given that the triangle with maximum area formed by a subset of the points is
$$a_1 a_p a_q$$
with $p, q \in \{2, 3, \ldots, 201\}$ find the sum of all possible values of $p + q$.
9. Find the area of the largest square than be inscribed within a regular octagon with sidelength 1.

10. Let p_0 be a square with sidelength 1. Let q_0 be the circle inscribed within p_0. For all positive integers n, let p_n for $n \geq 1$ be the square formed by connecting the midpoints of p_{n-1}. Let q_n be the circle inscribed within p_n. Let $[X]$ denote the area of figure X. Find the value
$$\sum_{i=1}^{\infty} |[p_i] - [q_i]| + \sum_{i=2}^{\infty} |[q_{i-1}] - [p_i]|.$$